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Boundary description of planckian scattering in curved spacetimes

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Abstract: We show that for an eikonal limit of gravity in a space-time of any dimension with a non-vanishing cosmological constant, the Einstein-Hilbert action reduces to a boundary action. This boundary action describes the interaction of shock-waves up to the point of evolution at which the forward light-cone of a collision meets the boundary of the space-time. The conclusions are quite general and in particular generalize work of E. and H. Verlinde. The role of the off-diagonal Einstein action in removing the bulk part of the action is emphasised. We discuss the sense in which our result is a particular example of holography and also the relation of our solutions in AdS to those of Horowitz and Itzhaki.

Keywords: AdS-CFT Correspondance, Models of Quantum Gravity, Black Holes.

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Contents

1. Introduction

Although one could claim that high-energy scattering in gravity should be treated in string theory the philosophy adopted in this article, based on the holographic principle is that such collisions should be treatable in the context of quantum gravity. The holographic principle is taken to be the guiding feature behind quantum gravity, rather than the string principle. As such it implies a reduction in the true number of quantum gravity degrees of freedom in line with the counting of degrees of freedom in string theory. Thus implementing correctly the holographic principle [2]–[5] in quantum gravity should result in a softening of amplitudes akin to that which occurs in string theory. From hereon all discussions will take place in the context of gravity using the Einstein-Hilbert action including cosmological constant apart from some string-theory related comments in the final sections.

The role of high energy scattering has been emphasized by 't Hooft in the context of the black hole evaporation process. As is well known, the appearance of Hawking radiation can be attributed to the enormous red-shift of outgoing wave packets when propagated back to the region close to the horizon. Quantum gravitational effects are therefore expected to play a fundamental role and their inclusion is expected to restore the unitarity of the Hawking radiation. According to the picture of 't Hooft these gravitational interactions close to the horizon can be effectively described by shock wave configurations associated to the boosted particles. They have non-trivial backreaction effects, bringing about a shift in the geodesics of the outgoing particles and in the position of the horizon. These correlations should in principle reduce the enormous degeneracy of states at the horizon of the black hole that one naively calculates using quantum field theory in the curved space-time of the near-horizon geometry. In this picture the horizon of the black hole becomes a sort of fluctuacting membrane due to incoming and outgoing particles and information of the bulk spacetime is projected holographically onto this surface.

In view of these developments it seems interesting to search for a more concrete relation between the general arguments of 't Hooft and Susskind and the AdS/CFT construction. In this paper we discuss in general the eikonal limit of scattering in curved space-times and find that under certain rather general assumptions about the relevant classical backgrounds, the dynamics of gravity are described by a theory that lives only on the boundary of the space-time. We also find that, from the bulk point of view, some of the classical solutions of this boundary theory describe shock-waves moving from the boundary to the bulk in Einstein spaces. On the way to finding classical backgrounds for our quantum theory and checking the classical limit of the theory we need the general solution of a two-dimensional gravity model analysed in [6]. Our solutions include the shock-wave solution constructed by Horowitz and Itzhaki [7] and this will be discussed in some detail in section 7.

Before proceeding, let us add that even if among our back-ground solutions we find black holes, in the eikonal regime one does not strictly expect black hole formation to appear. To describe the creation of small black holes one has to increase the energy transferred during the collisions. In other words, one has to go beyond the eikonal approximation. Black hole formation is an interesting issue which has been considered in a simplified $2+1$-dimensional set-up in [8]. Important related discussions in the context of the AdS/CFT correspondence and string theory can be found in [9, 10].

This paper is organised as follows. In section 2 we will describe the setup in which our analysis takes place in particular reviewing the basic idea of [1] in which a rescaling is made of the Einstein-Hilbert action thus separating it into three pieces each scaling differently in the eikonal limit. In section 3 we discuss the solutions to the classical part of this action in various regimes. In section 4 we introduce shock-wave configurations and then in section 5 we show how the off-diagonal part of the Einstein equations will be implemented. In section 6 we discuss the derivation and details of the resulting boundary action and in section 7 we show how our analysis is related to and extends the construction of Horowitz and Itzhaki [7]. Finally in section 8 we make some comments on our results and some other concluding remarks.

2. The setup

We consider high-energy scattering in spacetimes with a non vanishing cosmological constant Λ. Our basic construction is a direct generalization of that used in [1, 11] and thus we will consider an almost forward scattering situation. One introduces two scales, $\ell_{||}$ and $\ell_{\perp}$: the former is the typical longitudinal wavelength of the particles while the latter represents the impact parameter. Due to the presence of the cosmological constant we also have an additional scale ℓ – the radius of curvature $\Lambda \sim 1/\ell^2$. For high-energy forward scattering $\ell_{||}$ is typically of the order of the Planck length $\ell_{\rm Pl}$, $\ell_{\perp} \gg \ell_{||}$. This set of length scales characterizes the eikonal limit of the scattering process which for gravity is a linearized regime.[1] We will also deal with two different cases according to large or small values of the cosmological constant present in the problem. In general we then find that for $\ell_{\perp}$ small on the cosmological scale the scattering takes place in the locally flat space-time. On the other hand for impact parameters that are large on the cosmological scale, there are significant changes in the scattering process due to the curvature. The final result is conceptually the same however as we find that for shock-wave scattering the process can always be described by a lagrangian on the boundary at infinity of the space-time.

Our general strategy will be to choose dimensionless coordinates by extracting the natural length scale in the corresponding directions and therefore we will consider the Einstein-Hilbert action plus a non-vanishing cosmological constant and exterior curvature κ,

$$S = \frac{1}{{\ell_{\rm Pl}}^{d-2}} \left[\int_M \mathrm{d}^d x \sqrt{-G}\,(R - 2\Lambda) \right] + \frac{1}{\ell_{\rm Pl}^{(d-2)}} \int_{\partial M} \kappa \,, \tag{2.1}$$

[1] Note that as we are always considering planckian scattering energies such that $\ell_{||} \sim \ell_{\rm Pl}$ this coincides with the classical eikonal regime of $\ell_{||}\ell_{\perp} \gg \ell_{\rm Pl}^2$.

making a rescaling in the longitudinal x^α and transverse coordinates y^i according to the respective scales. Under a rescaling of the metric, the action rescales as

$$\epsilon^{d-4} S_E = \left(\frac{S_0}{\epsilon^2} + \frac{S_1}{\epsilon} + S_2 \right), \tag{2.2}$$

where $\epsilon = \ell_{\rm Pl}/\ell_\perp \sim \ell_{||}/\ell_\perp$ is a very small dimensionless parameter. $S_2 = S_{||}$ therefore is the strongly coupled part of the action while $S_0 = S_\perp$ is the weakly coupled part. The former is non pertubative while the latter is essentially classical. The role of S_1 will be discussed in the following but as is clear it also contributes to the classical part of the action in the limit of small ϵ. Under the above rescaling the cosmological term scales as $\ell_\perp^2/\epsilon^{d-4}$ and thus becomes part of the classical $S_\perp$ or the "quantum" $S_{||}$ depending on the size of $\ell_\perp$ in comparison to the cosmological scale, ℓ. We will consider both the case in which the cosmological constant is added to the classical part of the action — the "strongly curved regime" or the regime of large impact parameter — and the case when the cosmological constant is included in the strongly coupled part of the action — the "flat regime" or regime of small impact parameter.

2.1 Scaling and small fluctuations

We will actually consider a metric that at leading order is block diagonal — the blocks corresponding to the plane of the scattering and the plane transverse to the scattering. We will consider a rescaling of the metric (equivalent but more convenient than that of the coordinates discussed above) such that

$$G_{\mu\nu} = \begin{pmatrix} g_{\alpha\beta} & h_{\alpha i} \\ h_{i\alpha} & h_{ij} \end{pmatrix} \to \begin{pmatrix} \ell_{||}^2\, g_{\alpha\beta} & \ell_{||}\ell_\perp\, h_{\alpha i} \\ \ell_{||}\ell_\perp\, h_{i\alpha} & \ell_\perp^2\, h_{ij} \end{pmatrix}. \tag{2.3}$$

Greek indices $\mu, \nu, \ldots$ refer to all space-time coordinates whilst the indices $\alpha, \beta, \ldots$ refer only to the two-coordinates of the longitudinal scattering plane and the latin indices $i, j, \ldots$ refer to the directions transverse to the scattering plane. In addition to this rescaling of the energy scales, we will also make the assumption that the off-diagonal blocks of the metric are small. In the end then we will be making a double expansion of the action, in ϵ and in $h_{i\alpha}$.

In the limit that $\epsilon \to 0$ the leading terms in the action become classical and thus we need to derive and examine first the equations of motion arising from S_0 and S_1 given our choice of metric. S_0 always becomes a covariant $1+1$ dimensional action and has no terms linear in the small off-diagonal part of the metric $h_{i\alpha}$. S_1 starts at linear order in $h_{i\alpha}$ and the equation of motion here comes from the variation with respect to $h_{i\alpha}$ imposing the vanishing of the off-diagonal block of the Ricci tensor $R_{i\alpha}$. The remaining part S_2 of the action is the most interesting part as it is not removed in our limit and basically describes the dynamics of the eikonal limit of scattering at high-energy and large impact parameter. We will find that this action contains no bulk degrees of freedom and thus reduces to a boundary term. The details of the scaling of the curvature components are in appendix A. Taking into account the fact that $g^{\alpha\beta}$ scales as ϵ^{-2} relative to the scaling of g^{ij}, the order ϵ^{-2} term in the action has contributions from $R_{\alpha\beta}$ at order ϵ^0 and from R_{ij} at order ϵ^{-2}; the subleading order at ϵ^{-1} in the action comes solely from the leading term in $R_{i\alpha}$; while the final term at order ϵ^0 has contributions from the remaining lowest order terms in $R_{\alpha\beta}$ and R_{ij}. The resulting double expansion in ϵ and $h_{i\alpha}$ is;

$$\begin{aligned}
\epsilon^{d-4}\, S = &\frac{1}{\epsilon^2}\int_{\mathcal{M}} \sqrt{-gh}\, \left(R_g + \frac{1}{4}(h^{ik}h^{lm} - h^{il}h^{km})\partial_\alpha h_{ik}\partial_\beta h_{lm} g^{\alpha\beta}\right) - \\
&-\frac{2}{\epsilon}\int_{\mathcal{M}} \sqrt{-gh}\, h^{i\alpha} R_{i\alpha} + \\
&+\int_{\mathcal{M}} \sqrt{-gh}\, \left(R_h + \frac{1}{4}(g^{\alpha\beta}g^{\gamma\delta} - g^{\alpha\gamma}g^{\beta\delta})\partial_i g_{\alpha\beta}\partial_j g_{\gamma\delta} h^{ij}\right) + \\
&+\int_{\partial\mathcal{M}} \kappa - 2\ell_\perp^2 \int_{\mathcal{M}} \sqrt{-gh}\, \Lambda\,.
\end{aligned} \tag{2.4}$$

Considering a path integral for this action we see that the first two terms become classical as $\epsilon \to 0$. The cosmological constant can be moved to different orders of ϵ depending on it's scaling with $\ell_{||}$ or with $\ell_\perp$. Physically the mobility of the cosmological constant corresponds to the relationship between the scale of curvature of the space-time and the impact parameter of the scattering process under consideration. In the regime for which the curvature of the space-time does not really enter into the dicussion we find that the analysis is similar to that in flat space, though with corrections to the boundary action coming from the cosmological constant. In the other regime for which the impact parameter is larger than the radius of curvature, the space-time in the plane of scattering is curved and the analysis more subtle. The result again is that the scattering process can be described by a now non-quadratic lagrangian that lives on the boundary of the space-time.

The contribution of the exterior curvature will follow the usual construction of the Einstein-Hilbert action. It will split under rescaling to give contributions to the boundary in such a way that these boundary terms have their usual effect. That is, at the leading "classical" orders they will simply cancel boundary terms that come from integrating by parts when varying the action to get the equations of motion. The rescaling of the coordinates acts on the exterior curvature part of the action in such a way that it only contributes to the action at order ϵ^{-2} and ϵ^{-1} and thus will not provide any addition to our final boundary action which is at order ϵ^0. The details of the scaling of the exterior curvature part of the action is given in appendix A.

The general setup that is obtained via this rescaling of the action by the factor ϵ (which depends on the energy scales of the problem) is one in which we have an energy dependent action. This means that we are not considering a high energy process in a theory that is already defined, but rather we are using the high energy

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"eikonal" limit to define for us a new action that (hopefully) isolates the degrees of freedom that are important for the problem at hand. In particular, as we will see from the classical solutions that come from the small ϵ limit, the space-time splits into a $2 + (d - 2)$ configuration in which the two parts are coupled only through the constraint that the off-diagonal part of the curvature vanish. The interaction between the two parts of the space-time - that transversal and that longitudinal — is restricted by $R_{i\alpha} = 0$. Therefore in the case of large cosmological constant although one may be tempted to interpret this as a limit of small AdS_d it is not. It is more simply a case in which the separation of the shock-waves in the transverse part of the space-time is large, and the size of the AdS_2 in the longitudinal space corresponds to a large curvature. However this is not obviously the same as a scattering in say the context of string theory in an AdS_d with large curvature, though it does retain some of the important features.

In the next three sections we will discuss in turn each order in ϵ of this rescaled action.

3. The solutions

The geometry in the longitudinal plane of the scattering is determined by the saddle point of S_0. To find the general solution we have to generalize the class of vacuum configurations allowing among the various possibilities the presence of the cosmological constant. We then need to assume that in general the transverse metric depends on the longitudinal coordinates through a warp factor

$$h_{ij}(x^\mu) = e^{\chi(x,y)}\tilde{h}_{ij}(y)\,. \tag{3.1}$$

On the right hand side of this expression x refers to the coordinates $x^\alpha, x^\beta, \ldots$ of the scattering plane while y refers to the transverse coordinates $y^i, y^j, \ldots$. This ansatz can also be used to study radial scattering situations provided one chooses a time and a radial coordinate in the longitudinal directions. We will find the classical solutions by substituting this ansatz into the action. One could equivalently write down the general equations for the classical action and then of course substitute this ansatz to find the specific solutions. We will treat the $d = 3$ case separately due to various inconvenient factors of $(d - 3)$ in the following general analysis.

$$S_0 = S_\perp = \int_M \sqrt{-g\tilde{h}}\, e^{\frac{(d-2)\chi}{2}} \left(R[g] - 2\Lambda - \frac{(d-2)(d-3)}{4} g^{\alpha\beta}\partial_\alpha\chi\partial_\beta\chi \right)\,. \tag{3.2}$$

Making the following field redefinition

$$\phi(x,y) = \left(\frac{d-3}{2(d-2)}\right)^{1/2} \exp\left(\left(\frac{d-2}{4}\right)\chi(x,y)\right) \tag{3.3}$$

one gets

$$S_\perp = -8 \int_M \sqrt{-g\tilde{h}} \left(g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - \frac{(d-2)}{4(d-3)}\phi^2(R[g] - 2\Lambda) \right) . \tag{3.4}$$

The eikonal limit restricts us to consider the extrema of (3.4). It is interesting to note that with the assumption (3.1) the problem is reduced to a general two-dimensional gravity plus scalar field as studied in [6]. More properly, since the transverse fluctuations are suppressed in the leading order (in $h_{i\alpha}$) term of the weakly coupled action, its explicit expression will not contain, as shown by the scaling arguments, transverse derivatives. Therefore the action still depends on all four coordinates but the dependence on the transverse directions is only parametric. The equations of motion for the metric and the scalar field ϕ are:

$$\partial_\alpha\phi\partial_\beta\phi - \frac{1}{2}\, g_{\alpha\beta}g^{\gamma\delta}\partial_\gamma\phi\partial_\delta\phi = \frac{(d-2)}{4(d-3)}(\Lambda\, g_{\alpha\beta}\,\phi^2 + (g_{\alpha\beta}\Box - \nabla_\alpha\nabla_\beta)\phi^2) \tag{3.5}$$

$$\Box\phi + \frac{(d-2)}{4(d-3)}(R[g] - 2\Lambda)\phi = 0 \, . \tag{3.6}$$

As proved in the paper [6], all classical solutions have a Killing vector that is perpendicular to the curves of constant scalar field. Therefore for static configurations with $\Lambda < 0$ we can choose the metric to be of the form,

$$\mathrm{d}s^2 = -e(x)^2\mathrm{d}t^2 + g(x)^2\mathrm{d}x^2 \, , \tag{3.7}$$

where also

$$\phi = \phi(x) \, . \tag{3.8}$$

More properly in our case, as we will see below, e and g depend on the transverse coordinates too, since we are considering the two dimensional longitudinal manifold times the transverse space.

The static configurations are the ones relevant to the case of a negative cosmological constant. They have a boundary at spacelike infinity. Cosmological solutions relevant to de Sitter space have a spacelike boundary and are retrieved from the above solutions either by analytic continuation or by introducing an explicit time-dependent ansatz. The metric then becomes for $\Lambda > 0$,

$$\mathrm{d}s^2 = -g(t)^2\mathrm{d}t^2 + e(t)^2\mathrm{d}x^2 \, , \tag{3.9}$$

where now

$$\phi = \phi(t) \, . \tag{3.10}$$

These are in fact the type of solutions considered in [6].

3.1 Large curvature

For the case of a negative cosmological constant, the general solution to these equations can easily (details in appendix B) be found and is;

$$\begin{aligned} \phi(r) &= \psi(r)^{\gamma} \\ e(r) &= C\psi(r)^{\frac{\gamma}{4Q}}\dot{\psi}(r)\,, \end{aligned} \tag{3.11}$$

where

$$\begin{aligned} \psi(r) &= Ae^{\sqrt{\frac{\lambda}{4Q\gamma}}r} + Be^{-\sqrt{\frac{\lambda}{4Q\gamma}}r}\,, \\ \mathrm{d}r &= g(x)\mathrm{d}x\,, \end{aligned} \tag{3.12}$$

and

$$\gamma = \frac{4Q}{1+8Q} \tag{3.13}$$

and where $\lambda = -\frac{(d-2)}{2(d-3)}\Lambda$, $Q = \frac{(d-2)}{4(d-3)}$.

Notice that A, B and C are constant with respect to the longitudinal coordinates. However they can have an arbitrary dependence on the transverse coordinates y^i. Their precise form is fixed by imposing opportune boundary conditions depending on the spacetime under consideration.

It is also interesting to notice that if one takes either of A or B to zero, this two-dimensional metric has constant curvature and is actually just the metric on AdS_2 — the entire space-time metric being AdS_2 times the $(d-2)$-dimensional transverse geometry plus a warp factor.

3.2 Small curvature

In the small curvature regime, the cosmological constant term belongs to the strongly coupled part of the action. The classical action that we then have to consider is therefore (3.4) with $\Lambda = 0$. However, putting the cosmological constant to zero in the solutions is a singular limit. It is easy to directly solve for the metric in this case and one finds:

$$\begin{aligned} \phi(r) &= (Ar+B)^{\gamma} \\ e(r) &= C(Ar+B)^{\frac{\gamma}{4Q}}\,. \end{aligned} \tag{3.14}$$

Again, A, B and C are allowed to depend on the transverse coordinates. The curvature is

$$R[g] = \frac{16QA^2}{(1+8Q)^2(Ar+B)^2}\,. \tag{3.15}$$

Notice that it is always positive. In the limit $B \to \infty$ we recover flat space, which was not a solution of the equations of motion in the strong regime. In the region $r \ll B$, the space has locally positive constant curvature.

Note that there is also a degenerate flat space solution for which ϕ is constant and

$$e = Ar + B\,. \tag{3.16}$$

Even though these solutions could not be obtained directly from those with Λ non-zero by setting Λ to zero they can be obtained as near-horizon limits of those geometries, and this exactly corresponds to first shifting the coordinate r and then taking the limit of small cosmological constant.

The second case, (3.16), is the near-horizon geometry for the solutions of the previous section, for $B/A > 0$. Indeed, a simple coordinate transformation brings the near-horizon metric into the form of the Rindler metric (see appendix A). In the same way (3.14) is the near-horizon geometry in the case $B/A < 0$, and again a simple coordinate transformation brings it into the form of a Rindler type metric with singular horizon (again see appendix A).

3.3 Three-dimensional space-time

The above formulae are not directly applicable for $d = 3$, although one can obtain the equations of motion by carefully setting $d = 3$ in the above equations. The action in three dimensions is,

$$S_\perp = \int \sqrt{-g\tilde{h}}\,\phi^2(R[g] - 2\Lambda)\,, \tag{3.17}$$

where $\phi = e^{\chi/4}$. The equations of motion for the scalar field and the metric are [12]:

$$\begin{aligned} \nabla_\alpha\nabla_\beta\phi^2 - g_{\alpha\beta}\nabla^2\phi^2 - g_{\alpha\beta}\Lambda\phi^2 &= 0\,, \\ R[g] &= 2\Lambda\,, \end{aligned} \tag{3.18}$$

and so the space-time always has constant curvature. It is therefore not surprising that the only solution we find is AdS_2. These equations are totally symmetric under interchanges of ϕ and e, and under reflections $r \to -r$. Therefore the general solution is

$$\begin{aligned} \phi(r) &= A\,e^{r/2\ell} \\ e(r) &= B\,e^{-r/\ell}\,, \end{aligned} \tag{3.19}$$

where ℓ is the AdS radius. This solution corresponds to pure AdS, as expected, with a scalar field ϕ that vanishes at the boundary and has a singularity at the horizon.

When $\Lambda > 0$, we obtain 2-dimensional de Sitter space.

4. Gravity at high energy and shock waves

This section is a necessary digression into the shock-wave solutions to the classical part of our action. We need to understand the form of these shock-waves as they will motivate our final choice for the metric that we will use in the remaining non-classical part of our action. The physics in the bulk that can be described classically via these shock-waves will then be the physics that is encoded in the boundary action.

It turns out that scattering at planckian energies is dominated by the gravitational force. Therefore one should have a complete theory of quantum gravity to describe these processes. However already in the eikonal regime that we are considering one can use semiclassical methods to get useful information.

At leading order gravitational interactions can indeed be described by shock wave configurations — gravitational waves with a longitudinal impulsive profile. Essentially this is the gravitational field surrounding a particle whose mass is dominated by kinetic energy therefore representing a sort of massless regime of General Relativity [13]–[22].

Explicit solutions in general spacetimes and their physical effects have been described by Dray and 't Hooft [16], using the so called cut and paste method. To fix ideas, suppose we want to introduce a shock wave along $u = 0$ (in light cone coordinates). To this purpose, one first performs a cut

$$v \to v + \theta(u) f(x^i)\,,$$

where θ is the usual step function and f is a function only of the transverse coordinates x^i. Then one reabsorbs this shift with an opportune coordinate transformation to obtain a metric containing a Dirac delta function with support at $u = 0$.

To give an example, the gravitational field of a massless particle in flat spacetime can be described by a metric of the form,

$$ds^2 = -du dv - 4p \ln(\mid x^i \mid^2)\delta(u) du^2 + dx^i dx_i\,,$$

where p is the momentum of the massless particle. The physical effects of such configurations play a crucial role in 't Hooft's description of the evaporation of a black hole. We refer to [3] for a detailed account and references.

Choosing $x^\mu = (x^+, x^-, x^i)$ and placing the massless particle at $x^+ = x^i = 0$, a natural way to rewrite this metric is then

$$ds^2 = \partial_\alpha X^- dx^\alpha dx^+ + d^2 x^i$$

with

$$X^- = x^- + p\,\theta(x^+) \ln(\mid x^i \mid^2)\,.$$

This means that if we want to describe the scattering of two high energy particles before a collision takes place we must use the generalized shockwave configuration with the metric in the longitudinal plane being of the form,

$$ds^2 = \partial_\alpha X^a \partial_\beta X^b \eta_{ab} \mathrm{d}x^\alpha \mathrm{d}x^\beta\,,$$

thus allowing a pair shock-waves of the above type in both x^+ and in x^-. Here the SO$(1,1)$ X^a vectors can in principle depend on all space-time coordinates. These are the configurations studied in [1] and below we will generalize this construction to include the presence of curvature in the longitudinal plane.

5. The constraint and solution-ansatz

The second order in our expansion is quite simple. It is

$$-\frac{2}{\epsilon}\int \sqrt{-gh}\,h^{i\alpha}R_{i\alpha}\,. \tag{5.1}$$

As this is order ϵ^{-1} we also need to implement the corresponding equation of motion (as we did for the leading order in the previous section). In this order basically the equation of motion appears as a constraint $R_{i\alpha}=0$ on the general solutions.

Before implementing this constraint we will go back to the construction of [1] where it is shown how to change variables in a way that simplifies the following analysis. The saddle-point of the transverse part of the action S_0 gives the dominant vacuum field configurations. In the absence of the cosmological constant there was only

$$\begin{aligned} R[g] &= 0 \\ h_{ij} &= h_{ij}(y)\,. \end{aligned} \tag{5.2}$$

As recounted in the previous section for massless shock-wave configurations we will choose a parametrization of the metric via diffeomorphisms that represents these shock-waves,

$$g_{\alpha\beta} = \partial_\alpha X^a \partial_\beta X^b \eta_{ab}\,, \tag{5.3}$$

where the $X^a(x,y)$ are diffeomorphims which relate $g_{\alpha\beta}$ to the flat metric. Note that they are maps of the two dimensional x^α plane onto itself being however allowed to vary in the transverse directions and therefore represent transverse coordinate dependent displacements in the longitudinal coordinates. These X^a fields have the appearance of diffeomorphisms in the world-volume of the two-dimensional sigma-model and as such would appear to not introduce any new degrees of freedom. However, in the d-dimensional theory this is no longer really true as we are not considering the full transformation of the higher dimensional metric under these transformations. Nevertheless, due to the constraint coming from the off-diagonal part of the Einstein action we will see that these fields do not contribute additional bulk degrees of freedom.

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An intermediate and useful step required to derive the boundary action and used to great effect in [11] is to express the strongly coupled action in terms of fields V_i^α defined as

$$\partial_i X^a = V_i^\alpha \partial_\alpha X^a \,. \tag{5.4}$$

These fields were introduced and motivated physically in terms of fluid velocity in [11]. In the gravitational setup presented here they can be thought of as zweibeins (see also [23]) for a two-dimensional sigma-model describing the embedding of the scattering plane into the transverse space. They considerably simplify the action and help to conceptualize our configuration from the sigma-model point of view.

This definition could also have been motivated by the simple practical consideration that in order to rewrite the strongly coupled action as a boundary action one needs to remove derivatives in the transverse directions to give one an action that is covariant in the longitudinal directions. As a consequence one tries to express every derivative in the transverse directions in terms of a derivative in the longitudinal ones. This is precisely obtained utilizing this definition of the V_i^α fields. In this way the indices labelling transverse directions act as an internal symmetry of the sigma-model from the point of view of the longitudinal spacetime. This will become clear in the next section where we write the general explicit form for the boundary action for all $d \geq 3$ and in both the strong and weak curvature regimes.

This construction is basically identical for the more general metrics considered here. As we have seen in section 4, the conditions (5.2) are too restrictive and one ends up in this general setup case with a family of solutions specified by $g_{\alpha\beta}$ and χ. A natural generalization of the above parametrization of $g_{\alpha\beta}$ is then,

$$g_{\alpha\beta} = e^{\sigma(X)} \partial_\alpha X^a \partial_\beta X^b \eta_{ab} \,, \tag{5.5}$$

thus allowing the presence of a warp factor in the $2 + (d-2)$ decomposition of the metric. In principle σ may also have some explicit y dependence, however this would correspond to a more complicated sigma model than the one we are presently considering. As stated several times, the introduction of X is simply a statement that the scattering configuration described by shock-waves is described simply via singular co-ordinate tranformations with support only along light-cones in the scattering plane and thus the classical solution $\sigma(x)$ after the shock wave ansatz becomes simply $\sigma(X)$ and similarly $\chi(x)$ becomes $\chi(X)$. As in the previous case [11], we define fields V_i^α by

$$\partial_i X^a = V_i^\alpha \partial_\alpha X^a \,, \tag{5.6}$$

which in turn gives when lowering the longitudinal index

$$V_{i\alpha} = e^{\sigma(X)} \partial_i X^a \partial_\alpha X_a \,. \tag{5.7}$$

With the use of the $V_{i\alpha}$ the longitudinal metric changes under reparametrisations of the transverse coordinates according to,

$$\partial_i g_{\alpha\beta} = \nabla_\alpha V_{\beta i} + \nabla_\beta V_{\alpha i} \,. \tag{5.8}$$

Finally we also will have

$$h_{ij} = e^{\chi(X)} \tilde{h}_{ij}(y) \,. \tag{5.9}$$

Notice that this form of the solutions captures both the cases $\Lambda < 0$ and $\Lambda > 0$.

6. The effective boundary theory

We now examine how our classical solution – ansatz leads us to the general result that in this setup the transverse action $S_{\|}$ always reduces to a boundary action.

We now perform the substitution of our solution—ansatz into the leading order (ϵ^0) action,

$$\begin{aligned} S_{\|}[g,h] &= \int \sqrt{-gh}\left[R[h] - \frac{1}{4}\, h^{ij}\,\partial_i g_{\alpha\beta}\partial_j g_{\gamma\delta}\,\epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\right] \\ &= \int \sqrt{-gh}\left[R[h] - \epsilon_{\alpha\gamma}\epsilon_{\beta\delta}\, h^{ij}\nabla^\alpha V_i^\beta \nabla^\gamma V_j^\delta + \frac{1}{2} h^{ij} R_i R_j\right], \end{aligned} \tag{6.1}$$

where

$$R_i = \epsilon^{\alpha\beta}\nabla_\alpha V_{i\beta} \,. \tag{6.2}$$

6.1 Strong curvature regime

Filling in the solutions of the classical equations of motion,

$$\begin{aligned} g_{\alpha\beta} &= e^{\sigma(X)}\,\partial_\alpha X^a \partial_\beta X^b\,\eta_{ab} \\ h_{ij} &= e^{\chi(X)}\,\tilde{h}_{ij}\,, \end{aligned} \tag{6.3}$$

we get

$$\begin{aligned} S_{\|} = \int \sqrt{-g\tilde{h}}\, e^{\frac{d-4}{2}\chi}\Big[& R[\tilde{h}] - \epsilon_{\alpha\gamma}\epsilon_{\beta\delta}\tilde{h}^{ij}\nabla^\alpha V_i^\beta \nabla^\gamma V_j^\delta + \frac{1}{2}R_i^2 - \\ & - (d-3)\Box_{\tilde{h}}\chi - \frac{1}{4}(d-3)(d-4)(\partial_i\chi)^2\Big], \end{aligned} \tag{6.4}$$

and from now on we raise and lower transverse indices by means of the rescaled metric $\tilde{h}_{ij}$.

The action splits into a bulk and a boundary term:

$$
\begin{aligned}
S_{\|} &= S_{\rm bulk} + S_{\rm bdry} \\
&= \int_{\partial M} \mathrm{d}x^\alpha \sqrt{\tilde h}\, e^{\frac{d-4}{2}\chi} \Big(R[\tilde h]\, e^\sigma\, X^0 \partial_\alpha X^1 + e^\sigma \epsilon_{ab}\, \partial_i X^a \times \\
&\quad \times \Big[\partial^i \partial_\alpha X^b + \frac{1}{2} \nabla^\beta \sigma (\partial_\alpha X^b V^i_\beta - \partial_\beta X^b V^i_\alpha) \Big] - \frac{1}{2} V_{i\alpha} R^i - \frac{d-3}{2}\, \epsilon_{\alpha\beta} V^{i\beta} \partial_i \chi \Big) + \\
&\quad + \int_M \sqrt{-g\tilde h}\, e^{\frac{d-4}{2}\chi}\, V^{i\alpha} R_{i\alpha}\,, \qquad (6.5)
\end{aligned}
$$

where by X^a we mean the variation of X^a around its infinite value. Filling in the constraint

$$
\begin{aligned}
R_{i\alpha} &= \frac{1}{2} R[g] V_{i\alpha} + \frac{1}{2} \epsilon_{\alpha\beta} \nabla^\beta R_i + \frac{1}{2} \partial_\alpha \chi \nabla^\beta V_{i\beta} - \frac{d-3}{2} \partial_i \partial_\alpha \chi + \\
&\quad + \frac{d-4}{4} R_i\, \nabla^\beta \chi (\nabla_\alpha V_{i\beta} + \nabla_\beta V_{i\alpha}) \\
&= 0\,, \qquad (6.6)
\end{aligned}
$$

it obviously reduces to a boundary action. Note that this action will generally consist of two disconnected pieces corresponding to the two boundaries of the longitudinal space-time.

When $\Lambda < 0$, as we have been implicitly assuming in this section, the boundary is timelike. In the large curvature regime, the discussion for $\Lambda > 0$ is more intricate. Formally, the above derived action is valid in de Sitter space as well, the boundary being now a spacelike boundary at the future and past infinities. In this case the boundary theory is defined on an Euclidean manifold ∂M and thus the physical interpretation in terms of causality and locality of a corresponding holographic map is somewhat more mysterious than in the AdS case. Our derivation suggests that a certain set of observables in a holographic description of de Sitter space can be defined as correlation functions of a theory living on a space-like surface [24]

6.2 Weak curvature regime

In this regime there are two types of solutions, curved (singular) and flat. The action is the same as in the strong curvature regime, apart from an additional term proportional to the cosmological constant,

$$
\begin{aligned}
S_{\|}[g,h] &= \int \sqrt{-gh} \Big[R[h] - 2\Lambda - \frac{1}{4} h^{ij} \partial_i g_{\alpha\beta} \partial_j g_{\gamma\delta}\, \epsilon^{\alpha\gamma} \epsilon^{\beta\delta} \Big] \\
&= \int \sqrt{-gh} \Big[R[h] - 2\Lambda - \epsilon_{\alpha\gamma} \epsilon_{\beta\delta}\, h^{ij} \nabla^\alpha V^\beta_i \nabla^\gamma V^\delta_j + \frac{1}{2} h^{ij} R_i R_j \Big]\,. \qquad (6.7)
\end{aligned}
$$

Again filling in the general solutions we get:

$$
\begin{aligned}
S_{\|} &= S_{\rm bulk} + S_{\rm bdry} \\
&= \int_{\partial M} \mathrm{d}x^\alpha \sqrt{\tilde{h}} \Big((R[\tilde{h}] - 2e^\chi \Lambda) \;\; X^0 \partial_\alpha X^1 + e^{\frac{d-4}{2}\chi + \sigma} \epsilon_{ab}\, \partial_i X^a \times \\
&\quad \times \left[\partial^i \partial_\alpha X^b + \frac{1}{2} \nabla^\beta \sigma (\partial_\alpha X^b V^i_\beta - \partial_\beta X^b V^i_\alpha) \right] - e^{\frac{d-4}{2}\chi} \left(\frac{1}{2} V_{i\alpha} R^i + \frac{d-3}{2} \epsilon_{\alpha\beta} V^{i\beta} \partial_i \chi \right) \Big) + \\
&\quad + \int_M \sqrt{-g\tilde{h}}\, e^{\frac{d-4}{2}\chi}\, V^{i\alpha} R_{i\alpha}\,.
\end{aligned}
\tag{6.8}
$$

The most interesting case is the flat-space solution, where the action is simply quadratic:

$$
\begin{aligned}
S_{\|} &= S_{\rm bulk} + S_{\rm bdry} \\
&= \int_{\partial M} \mathrm{d}x^\alpha \sqrt{\tilde{h}} \left[\epsilon_{ab}\, X^a \left(\frac{1}{2} R[\tilde{h}] - \Lambda - \triangle_{\tilde{h}} \right) \partial_\alpha X^b - \frac{1}{2} V_{i\alpha} R^i \right] + \\
&\quad + \int_M \sqrt{-g\tilde{h}}\, e^{\frac{d-4}{2}\chi}\, V^{i\alpha} R_{i\alpha}\,.
\end{aligned}
\tag{6.9}
$$

The constraint then reads

$$
R_{i\alpha} = \frac{1}{2} \epsilon_{\alpha\beta} \nabla^\beta R_i = 0\,. \tag{6.10}
$$

Note that unlike [1] this does not imply that $R_i = 0$ but that R_i is a function only of the transverse coordinates $R_i(y)$. In particular then even for the flat space we have found that the complete analysis of this limit actually implies that there can be an additional term in the boundary action. It would be interesting to understand the physical meaning of this extra piece.

The action can be rewritten as

$$
\begin{aligned}
S_{\|} &= S_{\rm bdry} \\
&= \int_{\partial M} \mathrm{d}x^\alpha \sqrt{\tilde{h}} \left\{ \epsilon_{ab}\, \partial_\alpha X^a \left(\triangle_{\tilde{h}} + \Lambda - \frac{1}{2} R[\tilde{h}] \right) X^b + \frac{1}{2} \partial_\alpha X^a R^i(y) \partial_i X_a \right\}
\end{aligned}
\tag{6.11}
$$

which will be convenient for the discussions in the next section. Needless to say that in this case the classical solutions are independent of the value of the cosmological constant, and therefore the action (6.11) allows any value of Λ. We thus find ourselves with a quadratic action like that of [1]. Correspondingly there will be a way to quantize this action, write down the S-matrix and to study the inevitable non-commutativity of the boundary coordinates. In the next section we will consider the relationship between our construction and the curved space-time shock-wave scattering considered in particular in the paper of Horowitz and Itzhaki [7].

7. Shock-waves from eikonal gravity and the AdS/CFT correspondence

The boundary action found in the small curvature regime for $R_i(y) = 0$ is quadratic and therefore easy to deal with. In fact it is a straightforward generalisation of the boundary action found in [1, 11].

Let us briefly discuss its quantum mechanical properties when we couple it to point particles. In this regime, and restricting ourselves to the classical solutions of the equations of motion, the longitudinal space is flat. Therefore, the coupling to point particles in this case goes precisely along the lines of [11, section 5.1]. Hence we will not discuss this issue at length here. For details about the stress-energy tensor of a pointlike particle we refer the reader to appendix C.

Quantisation of the boundary action in the weak-coupling regime is straightforward and, as discussed in [11], it leads to non-trivial commutators for the coordinates X^a:

$$[X^a(y), X^b(y')] = i\epsilon^{ab} f(y, y')\,, \tag{7.1}$$

where f satisfies the Green's function equation

$$\left(\triangle_{\tilde{h}} + \Lambda - \frac{1}{2}\, R[\tilde{h}]\right) f(y, y') = \delta^{(d-2)}(y - y')\,. \tag{7.2}$$

As we have already discussed we expect shock-waves to also be described by our boundary action. Let us first briefly discuss how shock-waves can be implemented in AdS [7].

We write pure AdS in the following co-ordinates,

$$\mathrm{d}s^2 = \frac{4}{(1 - y^2/\ell^2)^2}\, \eta_{\mu\nu} \mathrm{d}y^\mu \mathrm{d}y^\nu, \tag{7.3}$$

where $y^2 = \eta_{\mu\nu} y^\mu y^\nu$. The stress tensor of a massless particle is computed in appendix C and gives

$$T_{uu} = -p\, \delta(u) \delta(\rho), \tag{7.4}$$

where ρ is the radial co-ordinate $\rho = \sum_{i=1}^{d-2} y_i^2$.

Horowitz and Itzhaki found the following solution of Einstein's equations with a massless particle:

$$\mathrm{d}s^2 = \frac{4}{(1 - y^2/\ell^2)^2}\, \left(\eta_{\mu\nu} \mathrm{d}y^\mu \mathrm{d}y^\nu + 8\pi G_{\mathrm{N}}\, p \delta(u)(1 - \rho^2/\ell^2) f(\rho) \mathrm{d}u^2\right) \tag{7.5}$$

provided

$$\triangle_h f - 4\, \frac{d-2}{\ell^2} f = \delta(\rho)\,. \tag{7.6}$$

$\triangle_h$ is the laplacian on the transverse hyperbolic space,

$$\mathrm{d}s^2 = \frac{\mathrm{d}\rho^2 + \rho^2 \mathrm{d}\Omega_{d-3}^2}{(1-\rho^2/\ell^2)^2}\,, \tag{7.7}$$

and therefore (7.6) takes the form

$$f'' + \frac{d-3+(d-5)\rho^2/\ell^2}{\rho(1-\rho^2/\ell^2)} f' - \frac{4(d-2)}{\ell^2(1-\rho^2/\ell^2)^2}\, f = \delta(\rho)\,. \tag{7.8}$$

The solutions to (7.6) are given by:

$$\begin{aligned}
\mathrm{AdS}_3: &\qquad f(\rho) = \frac{\ell}{2}(C+\theta(\rho))\,\sinh\log\left(\frac{\ell+\rho}{\ell-\rho}\right) + \ell D\cosh\log\left(\frac{\ell+\rho}{\ell-\rho}\right)\\
\mathrm{AdS}_4: &\qquad f(\rho) = C\,\frac{1+\rho^2/\ell^2}{1-\rho^2/\ell^2}\,\log(\rho/D) + \frac{2C}{1-\rho^2/\ell^2}\\
\mathrm{AdS}_5: &\qquad f(\rho) = \frac{C}{1-\rho^2/\ell^2}\left(\frac{1}{\rho}+\frac{6\rho}{\ell^2}+\frac{\rho^3}{\ell^4}\right) + \frac{D}{\ell}\,\frac{1+\rho^2/\ell^2}{1-\rho^2/\ell^2}\,,
\end{aligned} \tag{7.9}$$

D is an arbitrary constant to be determined by boundary conditions. C is a constant of order 1 that can be computed either by explicit computation or by matching with the Minkowski solutions. The shift function f of course behaves like the solutions for shock-wave in Minkowski space $f \sim \frac{1}{|x|^{d-3}}$ in the limit when the AdS radius divided by the impact parameter goes to infinity, $\ell/\rho \to \infty$. In fact the metric (7.5) was derived by boosting a black hole to the speed of light while sending its mass to zero and keeping its energy fixed.

Notice that for an Einstein space with negative curvature and curvature radius $\Lambda = -\frac{(d-1)(d-2)}{2\ell^2}$ the above general equation for the shift function derived via our boundary action method (7.2) reduces to the condition (7.6) found by Horowitz and Itzhaki precisely when the transverse space is euclidean AdS_{d-2}:

$$\mathrm{d}s^2 = 4\,\frac{\mathrm{d}\rho^2 + \rho^2\mathrm{d}\Omega_{d-3}^2}{(1-\frac{\rho^2}{\ell^2})^2}\,. \tag{7.10}$$

In this case, the transverse curvature is

$$R[\tilde{h}] = -\frac{(d-2)(d-3)}{\ell^2}. \tag{7.11}$$

The class of solutions to (7.2) is however much larger than only shock-waves in pure AdS. It allows for solutions where the transverse curvature is postitive, negative or zero, and the cosmological constant is also allowed to take positive values. In the limit $\Lambda \to 0$, all our results of course agree with the results found in [16].

It is not surprising that we find an approximate shock-wave from our boundary action only in the small curvature regime. These shock-waves have a smooth limit as $\Lambda \to 0$ which of course could not happen in the large curvature regime.

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Horowitz and Itzhaki have argued that the CFT duals of shock-waves are "light-cone states" — states with their energy-momentum tensor localised on the boundary light-cone. It is tempting to argue that our boundary description should somehow be related to these light-cone states. Indeed, we have shown that our boundary theory describes bulk shock-waves in an approximate fashion. Hence one is led to speculate that our boundary action is somehow related to some sort of eikonal limit of a boundary CFT perturbed by the addition of light-cone states. Notice, however, that it is not at all clear how to prove such a relation. In particular it is not clear how light-cone states should be precisely described in field theory, although some attempts have been made in [25]. Related discussions can be found in [26, 27, 28] and, recently, in [29]. Furthermore if quantum gravity has a boundary description at all energies, we have nevertheless taken the eikonal limit of it thereby explicitly breaking the possible covariance of the boundary theory. An interesting question is whether it is possible to do an eikonal approximation in a covariant way, or whether it is possible to restore covariance afterwards. Progress in the latter direction for the Minkowski case is found in [3, 30, 15]. In particular, in the simplified 2+1 – dimensional setup, restoring Lorentz covariance is tantamount to going beyond the extreme eikonal regime [15].

It would be extremely interesting if we could find an analog of (7.1) in the context of the AdS/CFT correspondence. This would amount to identifying the operators X^a in the CFT and to interpreting them from the bulk point of view. Based on previous considerations by 't Hooft and a computation of the trajectories of massless particles outlined in appendix C, they are expected to correspond to the positions of colliding particles, however a careful analysis is required in order to prove this. This could most easily be done using the techniques in [31] where boundary sources and operators are related to the coefficients of the perturbative expansion of bulk fields.

7.1 Boundary description of scalar fields in an AdS-shock-wave background

So far we have discussed single particles in an AdS background and interactions between quantum mechanical particles by means of shock-waves. One would however be ultimately interested in considering second quantised fields that interact gravitationally. In flat space, computing an S-matrix and extracting from it the amplitude for scattering between massless particles is a relatively straightforward task even if the interactions are gravitational [20, 3]. In AdS, however, things are much more complicated due to the presence of the timelike boundary and the impossibility to separate wavepackets. These problems can be sidestepped by imposing appropriate boundary conditions on the fields and ensuring that the S-matrix is unitary [32]. However, this is not possible for all the modes, and in the context of the AdS/CFT correspondence we are interested in considering both normalisable and non-normalisable modes. For other discussions of the AdS S-matrix, see [26, 27, 28]. In this paper we will not consider this issue, but rather concentrate on the CFT duals of scalar fields with generic boundary conditions.

As a first step towards considering the full quantum mechanics of scalar fields interacting gravitationally in AdS, we consider scalar fields on an AdS-shock-wave background. We concentrate on conformally coupled scalar fields. These have the nice property that their equation of motion is invariant under Weyl rescalings, up to a certain weight. The Klein-Gordon equation for these fields is

$$\left(\Box_G - \frac{d-2}{4(d-1)}\, R[G]\right) \phi(y) = 0\,, \tag{7.12}$$

and so in the AdS-shock-wave background they have mass $m^2 = -\frac{d^2-1}{4\ell^2}$. Note that in this subsection we use the convention of the AdS/CFT correspondence for which the boundary of the space-time is d-dimensional.

We perform a conformal transformation by which we remove the double pole of the metric:

$$\mathrm{d}s^2 = G_{\mu\nu}\, \mathrm{d}y^\mu \mathrm{d}y^\nu = \frac{1}{\Omega(y)^2}\, \bar{G}_{\mu\nu} \mathrm{d}y^\mu \mathrm{d}y^\nu\,, \tag{7.13}$$

G being the AdS-shocwkave metric (7.5). The Klein-Gordon equation transforms into

$$\bar{\Box}\, \bar{\phi}(y) = 0\,, \tag{7.14}$$

calculated in the metric $\bar{G}_{\mu\nu}$, and

$$\bar{\phi}(y) = \Omega^{\frac{1-d}{2}}\, \phi(y)\,. \tag{7.15}$$

There is no curvature term in (7.14) because $R[\bar{G}] = 0$. For the metric $\bar{g}_{\mu\nu}$, the laplacian factorises into a flat piece plus a shock-wave part,

$$\bar{\Box}\, \bar{\phi}(y) = \eta^{\mu\nu} \partial_\mu \partial_\nu \bar{\phi}(y) - p\, \delta(u)\, F(\rho)\, \partial_v^2 \bar{\phi}(y) = 0\,. \tag{7.16}$$

Equation (7.16) is difficult to solve in general due to the transverse derivatives but it can be readily solved in the eikonal approximation. A simple plane-wave solution is given by

$$\phi(y) = \Omega(y)^{\frac{d-1}{2}}\, \exp\left[ikv + ikp\, \theta(u)\, F(\rho)\right], \tag{7.17}$$

as one would expect from a computation of trajectories: the only effect of the shock-wave is a shift of the wave function over a distance given by the shift. The full solution gives, in the eikonal approximation,

$$\phi(y) = \Omega(y)^{\frac{d-1}{2}} \int \mathrm{d}^d k\, a(k)\, e^{ipk\, \theta(u) F(\rho) + ik_\mu y^\mu} + \text{c.c.}\,, \tag{7.18}$$

where $k_\mu^2 = 0$. Note that this sense of eikonal approximation is the same as in previous sections — all transverse derivatives are set to zero.

To interpret this classical field from the CFT point of view, it is easiest to go to Poincare co-ordinates where the boundary is at $r = 0$. The above field then has the following expansion [31]

$$\phi(r, x) = r^{\frac{d-1}{2}} \phi_0(x) + \cdots \tag{7.19}$$

as it approaches the boundary. This is the expected behaviour for a field of mass $m^2 = -\frac{d^2-1}{4\ell^2}$ interpreted as the source or as the dual operator.

For $-d^2/4 < m^2 < -d^2/4 + 1$, which includes the case at hand, there are two independent modes which give two possible quantisation schemes in AdS. Klebanov and Witten have argued [33] that the existence of two independent modes for fields in this mass range implies the existence of two conformal field theories dual to the same bulk metric. These are called the Δ_+ and the Δ_--theory. In the Δ_+-theory, the lowest-order in r mode, ϕ_0, has the usual interpretation as an external source that couples to an operator $O(x)$ of conformal dimension Δ_+, whereas $\varphi(x)$ (which appears at order Δ_-) is related to the expectation value of $O(x)$. In the Δ_--theory, on the other hand, ϕ_0 is interpreted as an expectation value whereas φ is the source. Both theories are related by a Legendre transformation. The case $\Delta_+ = \Delta_-$ is special and corresponds to the tachyon of minimal mass.

Let us consider the Δ_+-theory, where ϕ_0 corresponds to an operator of dimension $\Delta = \frac{d-1}{2}$,

$$\langle O(x) \rangle = -\phi_0(x) \,. \tag{7.20}$$

The expression for ϕ_0 can be obtained from (7.18). Notice that as $r \to 0$ the step function approaches $\theta(u) \to \theta(\frac{t^2-\vec{x}^2}{t})$. This means that the operator $O(x)$ has different expectation values on either side of the light-cone, $|t| > |\vec{x}|$ and $|t| < |\vec{x}|$, and furthermore there is a reflection as $t \to -t$. The operator acquires a certain "dressing" inside the light-cone. In the Δ_--theory, where ϕ_0 is interpreted as a source for $O(x)$, we see that the effect of the shock-wave is to introduce an explicit time-dependence in the source.

As pointed out in [7] shock-waves in AdS correspond to states with a stress-energy tensor concentrated on the light cone. We have found that when we also turn on a source for an operator of dimension $\Delta = \frac{d-1}{2}$ in the background of these light-cone states, the operator aquires different values on either side of the light-cone. Elaborating this a little bit further along the lines of [34] let us add that there is a map between the creation and the annihilation operators of the field ϕ and the composite operators in terms of which $O(x)$ is expanded. This however assumes a well-defined field theory for the scalar field ϕ in AdS, which we certainly have not constructed here (see however [32, 35]). One has to find a complete set of operators that generate the Hilbert space of the boundary theory and that have a well-defined inner product. This imposes additional conditions on the solutions (7.18) for them to be normalisable,

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like the quantisation of the frequencies. It would be most interesting to work out all these details, and to have an explicit field theory realisation of these phenomena.

The next step would be to consider gravitationally interacting fields in this AdS background. In reference [36] it was shown that fields interacting by means of shock-waves on a black hole horizon satisfy an exchange algebra, of the form:

$$\phi_{\rm out}(y)\phi_{\rm in}(x) = \exp\left[if^{ab}(|x-y|)\frac{\partial}{\partial x^a}\frac{\partial}{\partial y^b}\right]\phi_{\rm in}(x)\phi_{\rm out}(y)\,, \tag{7.21}$$

where $f^{ab} = \epsilon^{ab} f$. Here the non-commutativity of the fields was ascribed to the fluctuations of the horizon due to in-coming and out-going shock-waves. In reference [37], an alternative derivation of this exchange algebra is given for Minkowski space. The derivation does not use the presence of a horizon, but only the fact that creation operators create particles that carry shock-waves with them and thus produce shifts on the entire back-ground space-time. This is closely related to the form (7.18) of the solutions of the Klein-Gordon equation, which up to a conformal factor is the same in AdS and in Minkowski space. Therefore it seems reasonable to expect that this form of non-commutativity in the bulk due to shock waves produces some sort of "non-commutative" deformation of the commutative CFT living on the boundary of AdS. It would be interesting to find an interpretation of this deformation in terms of operators in the CFT. In performing such a derivation however we can no longer ignore the problem of correct quantisation of fields in AdS.

It seems likely that yet another way to derive the algebra (7.21) is by coupling our boundary action not to point particles but to scalar fields whose energy-momentum tensor is concentrated mainly in the longitudinal space.

It is interesting to note that shock-wave solutions are exact solutions of string theory. Indeed, in [38] it has been shown that shock-wave backgrounds are solutions to all orders in the sigma-model perturbation theory. In [7], it was shown also that the AdS shock-wave does not receive any α'-corrections using a geometrical argument [39, 40]. The argument uses the fact that all scalar combinations that can be formed from the contribution to the Riemann tensor due to the shock-wave vanish. Thus, corrections to the supergravity action can only come from the AdS part of the metric, but these are known to vanish. Thus, shock-waves are among the few known examples of exact backgrounds of string theory. Another interesting fact is that the amplitude computed by 't Hooft agrees, at large distances, with the amplitude of a free string in the shock-wave background generated by another string. So, the shock wave can be regarded as a non-perturbative effect coming from the resummation of flat-metric string contributions [14, 41]. At small distances, however, the string amplitudes do not exhibit the singular behaviour of the point particle case. Let us however point out that to our knowledge no amplitude valid beyond the eikonal regime has been computed so far for the point particle case, and so there is not much one can conclude from the discrepancy.

8. Comments and conclusions

Our analysis is a semi-classical analysis in the sense that we have setup a path-integral involving $S_{\parallel}$ that in addition involves only the fluctuations with insertion of fields all taking place on the boundary. Thus we have actually constructed a general proof of a particular form of holography — that corresponding to interactions of massless particles via gravitational shock-wave dynamics encoded in a theory of fluctuations on the boundary.

We would like to point out that our derivation requires no specific gauge choice. This agrees with [1] though not with the earlier paper [11]. However we do impose the requirement on our metric that it is of an approximately $2 + (d - 2)$ block-diagonal form with small off-diagonal components $h_{i\alpha}$. We have seen that in the course of our construction it was indeed very important to retain the small off-diagonal $h_{i\alpha}$ as it was precisely due to this that the constraint $R_{i\alpha}$ was derived and which was of importance to remove all bulk terms in the theory.

The fact that in this eikonal limit the theory becomes holographic in the sense described above is due not *only* to the fact that we treat essentially as a classical background the transverse metric but also to the crucial fact that one has an additional constraint to impose. As already remarked this constraint arises from the linear fluctuations in $h_{i\alpha}$ at order ϵ^{-1} in the rescaled action and is therefore associated to small off-diagonal pieces of the metric. In the end there is therefore no complete decoupling of transverse and longitudinal components of the metric as they are tied together by the non-trivial constraint $R_{i\alpha} = 0$ (6.6).

An alternative approach to high energy scattering has been advocated by Amati, Ciafaloni and Veneziano [14]. Using the regularized effective lagrangian proposed by Lipatov [42] they constructed an eikonal S-matrix that resums all semiclassical terms coming from the superstring approach. Their relevant gravitational degrees of freedom are the longitudinal modes and their "intermediate component". This is indeed reminiscent of the splitting discussed here into transverse and longitudinal modes. The difference lies in the fact that the intermediate component is momentum dependent (thus not purely transverse to the incoming beam) inducing a non-locality in the effective vertices. Complete decoupling as in the Verlinde case can be achieved in what they call the non-interacting limit. Our analysis suggests that the non-interacting limit cannot simply be taken without also taking into account precisely the off diagonal pieces.

The off-diagonal constraint essentially restricts the variations of our solutions in the transverse directions. This is where the dependence on the transverse direction is really taken into account. If follows that to effectively obtain a boundary theory one simply imposes this constraint on the transverse dynamics. We could rephrase the state of affairs by saying that Einstein gravity in the eikonal can be constrained to be a topological theory on a two-dimensional manifold embedded in d dimensions provided some conditions are imposed on the "lapse function" V which allows one to move from one plane to another by means of transverse deformations.

A certain similarity in the imposure of constraints on the deformations in the transverse directions is reminiscent of Bousso's [24] idea of holographic screens. In Bousso's case the projection of the information is controlled by the Raychauduri equation of classical general relativity. The latter describes the focussing properties of geodesics and a specific set of rules and terminology is introduced to define an appropriate notion of screen. In particular the Raychauduri equation describes the change along light-like geodesics of the expansion θ,

$$\theta = \frac{dA}{d\lambda}\,, \tag{8.1}$$

where A is the cross-sectional area defined by a set of geodesics nearby to the geodesic with affine parameter λ [43, 44]. Screens are placed at submanifolds perpendicular to the geodesics at the point where the expansion changes from increasing to decreasing. These and similar considerations match indeed with the idea of the holographic principle realized by means of a holographic projection. When one examines propagation on a classical background, as in our case, geodesics are not any more straight lines because of gravity and this has of course consequences when one wants to project onto a boundary. The result of this projection in general is a non-trivial boundary theory. We would like to point out that the translation of these *classical* bulk fluctuations into a projected description from the brane point of view is at the moment pretty much unclear despite some recent attempts [26, 27].

We also recall that as stressed by 't Hooft the gravitational interactions close to the horizon of a black hole or more generally at high energies are precisely described by shock wave configurations associated to boosted particles. They have non trivial backreaction effects, bringing about a shift in the geodesics of the outgoing particles which induces a form of non-commutativity at the quantum level. This has been observed in the analysis of [1] and similarly occurs here in the particular subset of cases considered for which the boundary action is quadratic.

Throughout the paper we have worked with the Einstein-Hilbert action without including higher curvature corrections. However our method is perfectly applicable for these higher order terms too, and in fact considering them is important when the energy is increased above $1/\ell_{\mathrm{Pl}}$. When embedding our theory in a specific string theory, one also has to include additional matter fields. Notice, however, that for the case $d = 5$ it should be straightforward to embed our results in string theory by condidering backgrounds with a constant dilaton and a covariantly constant self-dual 5-form compactified for example on an S^5. This is left for future research.

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A. Scaling of curvature

In this section we give the details of the final rescaled (up to lowest order in $h_{i\alpha}$, all orders in ϵ) Ricci tensor $R_{\mu\nu}$. From here one can simply check the expansion of the Einstein-Hilbert action. Higher orders in $h_{i\alpha}$ (quadratic at $1/\epsilon^2$ and at ϵ^0) are not necessary as we are not going to consider the fluctuations of the metric.

$$\begin{aligned} R_{\alpha\beta} \to\ & \epsilon^0 \left(R_{\alpha\beta} - \frac{1}{2}\nabla_\beta (g^{ik}\partial_\alpha g_{ik}) - \frac{1}{4} g^{ij}\partial_\alpha g_{kj} g^{km}\partial_\beta g_{im} \right) + \\ & +\epsilon^2 \left(-\frac{1}{2}\nabla_i (g^{ij}\partial_j g_{\alpha\beta}) - \frac{1}{4} g^{\gamma\rho}\partial_i g_{\gamma\rho} g^{ij}\partial_j g_{\alpha\beta} + \right. \\ & \left. + \frac{1}{4} g^{\gamma\rho}\partial_k g_{\beta\rho} g^{ki}\partial_i g_{\alpha\gamma} + \frac{1}{4} g^{\gamma\rho}\partial_k g_{\alpha\rho} g^{ki}\partial_i g_{\beta\gamma} \right). \end{aligned} \tag{A.1}$$

The leading term in $R_{i\alpha}$ is at zero order in $h_{i\alpha}$ which is already sufficient for our purposes as it is always multiplied by $h_{i\alpha}$ in the action and this term arises at order ϵ^0

$$\begin{aligned} R_{i\alpha} \to \epsilon^0 & \left(\frac{1}{2}\nabla_\beta (g^{\beta\rho}\partial_i g_{\alpha\rho}) - \frac{1}{2}\nabla_\alpha (g^{\beta\rho}\partial_i g_{\beta\rho}) + \frac{1}{2}\nabla_k (g^{kj}\partial_a g_{ij}) - \frac{1}{2}\nabla_i (g^{kj}\partial_\alpha g_{kj}) + \right. \\ & \left. + \frac{1}{4} g^{\gamma\rho}\partial_i g_{\alpha\rho} g^{kj}\partial_\gamma g_{kj} + \frac{1}{4} g^{km}\partial_\alpha g_{im} g^{\gamma\beta}\partial_k g_{\gamma\beta} - \frac{1}{2} g^{\beta\rho}\partial_\rho g_{ik} g^{kj}\partial_j g_{\alpha\beta} \right) \end{aligned} \tag{A.2}$$

R_{ij} is identical to $R_{\alpha\beta}$ under the interchange of Greek and Roman indices and $\epsilon \to \epsilon^{-1}$.

$$\begin{aligned} R_{ij} \to\ & \epsilon^{-2} \left(-\frac{1}{2}\nabla_\alpha (g^{\alpha\beta}\partial_\beta g_{ij}) - \frac{1}{4} g^{km}\partial_\alpha g_{km} g^{\alpha\beta}\partial_\beta g_{ij} + \right. \\ & \left. + \frac{1}{4} g^{km}\partial_\gamma g_{jm} g^{\gamma\alpha}\partial_\alpha g_{ik} + \frac{1}{4} g^{km}\partial_\gamma g_{im} g^{\gamma\alpha}\partial_\alpha g_{jk} \right) + \\ & +\epsilon^0 \left(R_{ij} - \frac{1}{2}\nabla_j (g^{\alpha\gamma}\partial_i g_{\alpha\gamma}) - \frac{1}{4} g^{\alpha\beta}\partial_\alpha g_{\gamma\beta} g^{\gamma\rho}\partial_j g_{\alpha\rho} \right). \end{aligned} \tag{A.3}$$

The exterior curvature part of the Einstein – Hilbert action is

$$S = \frac{1}{\ell_{\rm Pl}^{d-2}} \int \sqrt{K} \nabla_\mu n^\mu \tag{A.4}$$

K is the boundary metric which under rescaling is multiplied by $\ell_{||}^2 \ell_\perp^{2(d-2)}$. The normal n will have a non-zero component only in the direction perpendicular to the boundary, parallel to the longitudinal scattering plane. Thus as the longitudinal metric scales with $\ell_{||}^2$ the normalisation condition for n implies that it will also scale with $\ell_{||}$. Thus,

$$\nabla_\mu n^\mu = \frac{\nabla_\alpha n^\alpha}{\ell_{||}} + \frac{\nabla_i n^i}{\ell_\perp}\,. \tag{A.5}$$

The exterior curvature term of the action becomes

$$\epsilon^{d-4} S_{\partial M} = \frac{1}{\epsilon^2} \int \sqrt{K} \nabla_\alpha n^\alpha + \frac{1}{\epsilon} \int \sqrt{K} \nabla_i n^i\,. \tag{A.6}$$

As claimed in the text there is no additional contribution to the boundary action coming from the exterior curvature.

B. Classical solutions

In this appendix we give some more details on how to solve the equations of motion for the background, coming from the $1/\epsilon^2$ part of the action.

We rewrite the action (3.4) in the following form (now concentrating on the two-dimensional covariant part),

$$S = -\frac{1}{2} \int \sqrt{-g} \left(g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi + \Lambda \phi^2 - \frac{1}{2} \phi^2 R[g] \right). \tag{B.1}$$

This action belongs to the class of actions considered in [6], with lagrangian of the form

$$L = \sqrt{-g} \left(g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi - \lambda \phi^{2k} - Q \phi^2 R[g] \right), \tag{B.2}$$

with the obvious values $k = 1$, $\lambda = -\frac{(d-2)}{2(d-3)}\Lambda$, $Q = \frac{(d-2)}{4(d-3)}$.

As argued in the main text, we consider static metrics of the form (3.7). The lagrangian (with $k = 1$) then reduces to the particle lagrangian

$$L = \frac{1}{g} \left(e \phi'^2 - 4Qe'\phi\phi' \right) - \lambda g e \phi^2\,. \tag{B.3}$$

The prime denotes derivatives with respect to x. It is obvious that the field g does not contribute to the dynamics - the equation of motion for g is simply an expression of reparameterization invariance in the spatial co-ordinate. In fact, all the g-depenence disappears from the equations of motion if we define a new variable $r = \int_0^x \mathrm{d}x'\, g(x')$. We then get

$$\begin{aligned} -2Q\,\frac{\ddot{e}}{e} + \frac{\dot{\phi}\dot{e}}{\phi e} + \frac{\ddot{\phi}}{\phi} + \lambda = 0 \\ \frac{\ddot{\phi}}{\phi} + \left(1 + \frac{1}{4Q}\right)\left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{\lambda}{4Q} = 0 \\ \frac{\dot{\phi}}{\phi}\left(\frac{\dot{\phi}}{\phi} - 4Q\frac{\dot{e}}{e}\right) + \lambda = 0\,, \end{aligned} \tag{B.4}$$

and the dots denote derivatives with respect to r.

Substituting this solution one has for the curvature

$$R[g] = -2a^2\left[1 + \frac{3\gamma}{4Q} + \frac{\gamma(\gamma-4Q)}{16Q^2}\left(\frac{A - Be^{-2ar}}{A + Be^{-2ar}}\right)^2\right], \tag{B.5}$$

where

$$a^2 = \frac{\lambda}{4Q\gamma}\,. \tag{B.6}$$

We see from (B.5) that among various solutions we also have the case in which the curvature is constant if either $A = 0$ or $B = 0$.

Since both cases differ only by a coordinate transformation, we choose $B = 0$. The longitudinal metric $g_{\alpha\beta}$ is then

$$\mathrm{d}s^2 = -(aCA^q)^2 e^{2aqr}\mathrm{d}t^2 + \mathrm{d}r^2\,, \tag{B.7}$$

where

$$q = 1 + \frac{\gamma}{4Q} \tag{B.8}$$

This is indeed the AdS_2 metric with the proper warp factor growing linearly in the radial coordinate. Our coordinates, however, do not cover the whole of AdS. One finds global coordinates by defining

$$e^{aqr} = \cos\rho\,, \tag{B.9}$$

where $0 \le \rho \le \pi/2$.

The curvature $R[g] = -\frac{2\ddot{e}}{e}$ obviously simplifies and becomes

$$R[g] = -\frac{\lambda(4Q+\gamma)}{8Q^2\gamma}\,. \tag{B.10}$$

Furthermore, those solutions with A and B non-zero will be analogous in structure to AdS_2/Schwarzschild geometries, though the metric will have a different functional form due to the presence of the non-trivial scalar field. The mass of the configuration will be proportional to B.

In $d = 3$ there are small modifications due to the appearence of several $(d - 3)$ factors in the general solutions. We can easily proceed here as follows. The one-dimensional form of the action is:

$$L = \frac{e'\phi^{2\prime}}{g} - \Lambda e g \phi^2 \,, \tag{B.11}$$

and so the equations of motion reduce to

$$\begin{aligned} \ddot{\phi^2} + \Lambda\phi^2 &= 0 \\ \ddot{e} + \Lambda e &= 0 \\ \frac{\dot{\phi^2}\dot{e}}{\phi^2 e} + \Lambda &= 0, \end{aligned} \tag{B.12}$$

after reabsorbing the non-dynamical field g in the definition of the parameter r, as before.

B.1 Global structure of the solutions

The metric

$$\mathrm{d}s^2 = -e(r)^2\mathrm{d}t^2 + \mathrm{d}r^2 \tag{B.13}$$

has a horizon when $e(r) = 0$. There are two possible locations of this horizon, depending on the relative sign of the initial conditions A and B.

For $B/A > 0$, $e(r)$ has a simple zero. With the following rescalings of the coordinates,

$$\begin{aligned} r &= \sqrt{\frac{Q\gamma}{\lambda}}\log B/A + \eta \\ t &= \frac{4Q\gamma}{C\lambda}(4AB)^{-\gamma/8Q-1/2}\tau \end{aligned} \tag{B.14}$$

the metric near the horizon is simply the Rindler space metric,

$$\mathrm{d}s^2 = -\eta^2\mathrm{d}\tau^2 + \mathrm{d}\eta^2, \tag{B.15}$$

and so locally the space is flat.

For $B/A < 0$, we rescale the coordinates as follows:

$$\begin{aligned} r &= \sqrt{\frac{Q\gamma}{\lambda}}\log|B/A| + \eta \\ t &= \left(\frac{\lambda|AB|}{Q\gamma}\right)^{-\gamma/8Q-1/2}\tau\,, \end{aligned} \tag{B.16}$$

and we find the metric

$$ds^2 = -\eta^{\gamma/2Q} d\tau^2 + d\eta^2 \tag{B.17}$$

with curvature $R = -\frac{\gamma(\gamma-4Q)}{8Q^2\eta^2}$.

C. Null geodesics in AdS and the stress-energy tensor

The stress-energy tensor induced by a massive particle travelling along a path γ described by the trajectory $z^\mu(s)$ is given by

$$T^{\mu\nu}(x) = -\frac{p}{\sqrt{-G(x)}} \int_\gamma ds\, \delta^d(x - z(s))\, \dot{z}^\mu \dot{z}^\nu , \tag{C.1}$$

with the usual normalisation for delta-functions. p is the momentum of the particle along the light-cone.

In order to be able to use (C.1) for practical purposes, one has to compute the trajectories $z^\mu(s)$ with some boundary conditions. We will concentrate on trajectories of particles the momentum of which has components only in the v-direction, but it is a straightforward exercise to consider other cases.

It is well-known that the null geodesics of two conformally related space-times are the same up to a reparametrisation of the geodesic length. Therefore null trajectories in the above co-ordinates will take the same form as those in Minkowski space. It is nevertheless convenient for the computation of the stress-energy tensor to see explicitly how the affine parameter changes.

The geodesic equation and the mass-shell condition give

$$\begin{aligned} \frac{d}{d\lambda}\left(\frac{\eta_{\mu\nu}\dot{z}^\nu}{\Omega^2}\right) &= 2\frac{\eta_{\mu\nu} z^\nu \mathcal{L}}{\ell^2\, \Omega} \\ \mathcal{L} &= \frac{1}{\Omega^2}\, \eta_{\mu\nu}\dot{z}^\mu \dot{z}^\nu = 0\,. \end{aligned} \tag{C.2}$$

$\mathcal{L}$ is the Lagrange density defined by the second of (C.2), and λ the affine parameter along the geodesic. These equations integrate to

$$\eta_{\mu\nu}\, \dot{z}^\nu = v_\mu\, \Omega^2\,. \tag{C.3}$$

v_μ is a constant, lightlike vector satisfying $\eta^{\mu\nu} v_\mu v_\nu = 0$ to be determined by the boundary conditions. This equation also relates the affine parameter in AdS to the affine parameter in Minkowski space.

The stress-energy tensor of this particle now equals:

$$T_{\mu\nu} = -p\, \Omega^d\, v_\mu v_\nu \int ds\, \delta^d(y - z(s))\,, \tag{C.4}$$

and choosing co-ordinates where momentum is purely in the v-direction, this reduces to

$$T_{uu} = -p\,\Omega^d\,\delta(u-u_0)\,\delta(\rho-\rho_0)\,, \tag{C.5}$$

where $\rho = \sum_{i=1}^{d-2} y_i^2$. Notice that in order for the metric (7.5) to be a solution of Einstein's equations with this stress-energy tensor we need the initial condition $u_0 = 0$. It is also convenient to take $\rho_0 = 0$. Thus we get the stress-energy tensor,

$$T_{uu} = -p\delta(u)\delta(\rho). \tag{C.6}$$

Once one has (C.6), one can compute the back-reaction on the AdS metric, obtaining the solution found by Horowitz and Itzhaki. The next step is then to compute the geodesics of a test particle in the back-reaction corrected metric. The computation goes along the same lines as the one above. We do not give the details here since it is a straightforward exercise, but give only the results. We concentrate on trajectories whose initial velocities are perpendicular to the velocity of the shockwave, that is, the geodesics with $v = y^i = 0$ before the collision. This gives a head-on collision.

It turns out that the geodesic equations can again be exactly integrated, and the effect is the same as in Minkowski space: there is a shift in the v coordinate and a deflection in the x^i-plane which nevertheless is negligible in the eikonal approximation where the impact parameter is much larger than the Planck length. In this approximation, the shift is given by

$$\delta v = -8\pi G_{\rm N}\,p_u\,F_0\,\theta(u)\,, \tag{C.7}$$

where F_0 is the shift function before the collision, $F_0 = F(u=0)$.

Of course the same results can be found from geodesics in Minkowski space by noting that massless geodesics are invariant under conformal transformations of the metric.

It is interesting to note that, when one considers only one particle, there is no self-interaction, and therefore the present solution to the Einstein-matter system with the given boundary conditions is exact. However, when considering two particles this is not true anymore, and one has to restrict oneself to consider a "soft" test particle in the background of a "hard" particle.

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